[a] $(xy - x^3y^{-1})y' + 3x^2 = y^2$ **FINAL ANSWER**: $\frac{1}{2}x^{-2}y^2 - \ln|y| + 3\ln|x| = C$

SOLUTION 1 (exact using integrating factor in x) **GRADE AGAINST ONE SOLUTION ONLY**

$(3x^2 - y^2)dx + (xy - x^3y^{-1})dy = 0$

$$\frac{M_y - N_x}{N} = \frac{(-2y) - (y - 3x^2y^{-1})}{xy - x^3y^{-1}} = \frac{-3y + 3x^2y^{-1}}{xy - x^3y^{-1}} = \frac{-3(y - x^2y^{-1})}{x(y - x^2y^{-1})} = -\frac{3}{x}$$

which is a function of only x

$\mu = e^{\int -\frac{3}{x}dx} = e^{-3\ln|x|} = x^{-3}$

ALL UNDERLINED ITEMS ON ALL QUESTIONS WORTH 1 POINT, UNLESS OTHERWISE INDICATED

$(3x^{-1} - x^{-3}y^2)dx + (x^{-2}y - y^{-1})dy = 0$

CHECKPOINT: $M_y = -2x^{-3}y = N_x$, so equation is exact

$f = \int (x^{-2}y - y^{-1})dy = \frac{1}{2}x^{-2}y^2 - \ln|y| + C(x)$

$f_x = -x^{-3}y^2 + C'(x) = 3x^{-1} - x^{-3}y^2$

NO PARTIAL CREDIT FOR INCORRECT WORK AFTER CHECKPOINTS IF THE CHECKPOINT IS MISSING, FALSIFIED OR IGNORED

$\frac{1}{2}$ $C'(x) = 3x^{-1}$ CHECKPOINT: function of only x $\frac{1}{2}$

$C(x) = 3\ln|x|$

$\frac{1}{2}x^{-2}y^2 - \ln|y| + 3\ln|x| = C$

SOLUTION 2 (exact using integrating factor of form x^ay^b)

$(3x^2 - y^2)dx + (xy - x^3y^{-1})dy = 0$

$\frac{1}{4}$ $(3x^{a+2}y^b - x^ay^{b+2})dx + (x^{a+1}y^{b+1} - x^{a+3}y^{b-1})dy = 0$

$\frac{1}{4}$ $M_y = 3bx^{a+2}y^{b-1} - (b+2)x^ay^{b+1}$

$\frac{1}{4}$ $N_x = (a+1)x^ay^{b+1} - (a+3)x^{a+2}y^{b-1}$

$\frac{1}{4}$ $3b = -(a+3)$ and $-(b+2) = a+1 \Rightarrow 2b - 2 = -2 \Rightarrow b = 0 \Rightarrow a = -3 \Rightarrow \mu = x^{-3}$

$(3x^{-1} - x^{-3}y^2)dx + (x^{-2}y - y^{-1})dy = 0$

CHECKPOINT: $M_y = -2x^{-3}y = N_x$, so equation is exact

$f = \int (x^{-2}y - y^{-1})dy = \frac{1}{2}x^{-2}y^2 - \ln|y| + C(x)$

$f_x = -x^{-3}y^2 + C'(x) = 3x^{-1} - x^{-3}y^2$

$\frac{1}{2}$ $C'(x) = 3x^{-1}$ CHECKPOINT: function of only x $\frac{1}{2}$

$C(x) = 3\ln|x|$

$\frac{1}{2}x^{-2}y^2 - \ln|y| + 3\ln|x| = C$

SOLUTION 3 (homogeneous)

$(3x^2 - y^2)dx + (xy - x^3y^{-1})dy = 0$

$\frac{1}{2}$ $M(tx, ty) = 3(tx)^2 - (ty)^2 = 3t^2x^2 - t^2y^2 = t^2(3x^2 - y^2) = t^2M(x, y)$

$\frac{1}{2}$ $N(tx, ty) = (tx)(ty) - (tx)^3(ty)^{-1} = t^2xy - t^2x^3y^{-1} = t^2(xy - x^3y^{-1}) = t^2N(x, y)$

both homogeneous of degree 2

$y = vx \Rightarrow dy = v\,dx + x\,dv$

$(3x^2 - v^2x^2)dx + (vx^2 - v^{-1}x^2)(v\,dx + x\,dv) = 0$

$(3 - v^2)dx + (v - v^{-1})(v\,dx + x\,dv) = 0$

$2dx + (v - v^{-1})x\,dv = 0$

$\int \frac{2}{x}dx = \int (v^{-1} - v)\,dv$

$2\ln|x| + C = \ln|v| - \frac{1}{2}v^2$

$2\ln|x| + C = \ln|\frac{y}{x}| - \frac{y^2}{2x^2}$

$3\ln|x| + C = \ln|y| - \frac{y^2}{2x^2}$

$\frac{1}{2}x^{-2}y^2 - \ln|y| + 3\ln|x| = C$

OR

$x = vy \Rightarrow dx = v\,dy + y\,dv$

$(3v^2y^2 - y^2)(v\,dy + y\,dv) + (vy^2 - v^3y^2)dy = 0$

$(3v^2 - 1)(v\,dy + y\,dv) + (v - v^3)dy = 0$

$(3v^2 - 1)y\,dv + 2v^3dy = 0$

$\int \frac{2}{y}dy = \int (v^{-3} - 3v^{-1})\,dv$

$2\ln|y| + C = -\frac{1}{2}v^{-2} - 3\ln|v|$

$2\ln|y| + C = -\frac{y^2}{2x^2} - 3\ln|\frac{x}{y}|$

$C = -\frac{y^2}{2x^2} - 3\ln|x| + \ln|y|$

$\frac{1}{2}x^{-2}y^2 - \ln|y| + 3\ln|x| = C$

GRADE ONLY AGAINST THE SOLUTION WHICH MATCHES YOUR SUBSTITUTION

$y = vx$ OR $x = vy$

[b] $2(r+e^{-\theta}\sqrt{r}-r\tan\theta)d\theta-(\tan\theta)dr=0$ **FINAL ANSWER:** $r=e^{-2\theta}(1+C\sin\theta)^2$

$$(\tan\theta)\frac{dr}{d\theta}+2(\tan\theta-1)r=2e^{-\theta}\sqrt{r}$$

$1\frac{1}{2}$ $$\frac{dr}{d\theta}+2(1-\cot\theta)r=2e^{-\theta}\sqrt{r}\cot\theta$$

$$v=r^{1-\frac{1}{2}}=r^{\frac{1}{2}} \Rightarrow r=v^2 \Rightarrow \frac{dr}{d\theta}=2v\frac{dv}{d\theta}$$

$$2v\frac{dv}{d\theta}+2(1-\cot\theta)v^2=2e^{-\theta}v\cot\theta$$

$$\frac{dv}{d\theta}+(1-\cot\theta)v=e^{-\theta}\cot\theta$$

$$\mu = e^{\int(1-\cot\theta)d\theta} = e^{\theta-\ln|\sin\theta|} = e^{\theta}\csc\theta$$

$$e^{\theta}\csc\theta\frac{dv}{d\theta}+e^{\theta}\csc\theta(1-\cot\theta)v=\csc\theta\cot\theta$$

CHECKPOINT: $(e^{\theta}\csc\theta)' = e^{\theta}\csc\theta-e^{\theta}\csc\theta\cot\theta = e^{\theta}\csc\theta(1-\cot\theta)$

$$(e^{\theta}\csc\theta)v=-\csc\theta+C$$

$\frac{1}{2}$ $$v=-e^{-\theta}+Ce^{-\theta}\sin\theta$$

$$r^{\frac{1}{2}}=-e^{-\theta}+Ce^{-\theta}\sin\theta$$

$$r=e^{-2\theta}(-1+C\sin\theta)^2=e^{-2\theta}(1+C\sin\theta)^2$$

[c] $\dfrac{ds}{dt} = \dfrac{3st^2}{s^3 + t^3 - st^3}$ **FINAL ANSWER:** $-s^{-1}e^s t^3 + (s-1)e^s = C$

$(s^3 + t^3 - st^3)ds - 3st^2 dt = 0$

$\dfrac{M_t - N_s}{N} = \dfrac{(3t^2 - 3st^2) - (-3t^2)}{-3st^2} = \dfrac{6t^2 - 3st^2}{-3st^2} = -\dfrac{2}{s} + 1$, which is a function of only s

$\mu = e^{\int(-\frac{2}{s}+1)ds} = e^{-2\ln|s| + s} = s^{-2}e^s$

$(se^s + s^{-2}e^s t^3 - s^{-1}e^s t^3)ds - 3s^{-1}e^s t^2 dt = 0$

CHECKPOINT: $M_t = 3s^{-2}e^s t^2 - 3s^{-1}e^s t^2 = N_s$, so equation is exact

$f = \int -3s^{-1}e^s t^2 dt = -s^{-1}e^s t^3 + C(s)$

$f_s = s^{-2}e^s t^3 - s^{-1}e^s t^3 + C'(s) = se^s + s^{-2}e^s t^3 - s^{-1}e^s t^3$

$\frac{1}{2}$ $C'(s) = se^s$ CHECKPOINT: function of only s $\frac{1}{2}$

$C(s) = (s-1)e^s$

$-s^{-1}e^s t^3 + (s-1)e^s = C$